Exercise 12

For the following exercises, use long division to divide. Specify the quotient and the remainder.

$$(x^3 - 3x^2 + 5x - 6) \div (x - 2)$$

Solution

Set up the division problem, writing out every term in the dividend.

$$x - 2 \overline{\smash{)}x^3 - 3\,x^2 + 5\,x - 6}$$

Divide the leading term of the dividend by the leading term of the divisor and place the result above the term with the same power of x.

$$\begin{array}{r} x^2 \\ x - 2 \overline{\smash{)}x^3 - 3\,x^2 + 5\,x - 6} \end{array}$$

Multiply this result by the divisor and subtract it from the dividend.

$$\begin{array}{r} x^2 \\ x - 2 \overline{\smash{)}x^3 - 3\,x^2 + 5\,x - 6} \\ \underline{-\left(x^3 - 2\,x^2\right)} \\ -x^2 \end{array}$$

Bring the next term in the dividend down.

$$
\begin{array}{r}
x^2 \\
x-2\ \overline{\smash{)}x^3 - 3\,x^2 + 5\,x - 6} \\
\underline{-\left(x^3 - 2\,x^2\right)}\ \big\downarrow \\
-x^2 + 5\,x
\end{array}
$$

Divide the leading term of this modified dividend by the leading term of the divisor and place the result above the term with the same power of x.

$$
\begin{array}{r}
x^2 - x \\
x-2\ \overline{\smash{)}x^3 - 3\,x^2 + 5\,x - 6} \\
\underline{-\left(x^3 - 2\,x^2\right)} \\
-x^2 + 5\,x
\end{array}
$$

Multiply this result by the divisor and subtract it from the modified dividend.

$$
\begin{array}{r}
x^2 - x \\
x-2\ \overline{\smash{)}x^3 - 3\,x^2 + 5\,x - 6} \\
\underline{-\left(x^3 - 2\,x^2\right)} \\
-x^2 + 5\,x \\
\underline{-\left(-x^2 + 2\,x\right)} \\
3\,x
\end{array}
$$

Bring the next term in the dividend down.

$$
\begin{array}{r}
x^2 - x \\
x - 2 \overline{\smash{)}\, x^3 - 3\,x^2 + 5\,x - 6} \\
-\left(x^3 - 2\,x^2\right) \\
\hline
-x^2 + 5\,x \\
-\left(-x^2 + 2\,x\right) \\
\hline
3\,x - 6
\end{array}
$$

Divide the leading term of this modified dividend by the leading term of the divisor and place the result above the term with the same power of x.

$$
\begin{array}{r}
x^2 - x + 3 \\
x - 2 \overline{\smash{)}\, x^3 - 3\,x^2 + 5\,x - 6} \\
-\left(x^3 - 2\,x^2\right) \\
\hline
-x^2 + 5\,x \\
-\left(-x^2 + 2\,x\right) \\
\hline
3\,x - 6
\end{array}
$$

Multiply this result by the divisor and subtract it from the modified dividend.

$$
\begin{array}{r}
x^2 - x + 3 \\
x - 2 \overline{\smash{)}\, x^3 - 3\,x^2 + 5\,x - 6} \\
-\left(x^3 - 2\,x^2\right) \\
\hline
-x^2 + 5\,x \\
-\left(-x^2 + 2\,x\right) \\
\hline
3\,x - 6 \\
-(3\,x - 6) \\
\hline
0
\end{array}
$$

There are no further terms in the dividend to drop down, so the division is complete. The quotient is $x^2 - x + 3$, and the remainder is 0.

$$
(x^3 - 3x^2 + 5x - 6) \div (x - 2) = x^2 - x + 3
$$